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FACULTY WORKING
PAPER NO. 1162

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ABSTRACT

The purpose of this paper has been to improve the interpretation and forecasting of individual implied standard deviations for call options on SP500 futures. By empirically explaining their composition through time series analysis and cross-sectional time series regression models, and profitably utilizing this information to identify mispriced options, we have demonstrated the disadvantages to using individual option ISD measures. More importantly, our results provide some evidence as to how the Black option pricing model (and relatedly the Black-Scholes model) might be misspecified, or jointly, how the market might be inefficient.

A. Introduction

The derivation and use of the Implied Standard Deviation (ISD) for an option as originated by Latané and Rendleman (1976) has become the state-of-the-art methodology for variance estimation. Yet the fact that there will exist as many ISD's for an underlying asset as there are options on it, as well as their observable nonconstant nature, has attracted considerable attention from practitioner and theoretician alike.

From the practitioner's point of view the question has been one of how to use the ISD for determining mispriced options, assuming it represents the market's estimate of future volatility. For the academician, the inconsistent cross-sectional and time series nature of the ISD implies a certain and perhaps significant degree of misspecification within the Black and Scholes (B-S) option pricing model (and relatedly the Black option pricing model). Neither domain up to this point has pursued a careful investigation of these two issues in an integrated manner.

The focus of this paper will be to improve upon the forecast of the ISD while simultaneously explaining what this variable really symbolizes. From our results we will attempt to draw specific implications as to how the Black OPM (and relatedly the B-S model) might be misspecified.

Although a theoretical push to understand the nonstationary nature of the ISD has been gaining momentum for some time now, it is only with the advent of index options and options on index futures in 1983 and 1982 respectively, that an ample amount of option data has existed for a more thorough study of cross-sectional effects. This paper uses 1983 and 1984 data for the call option on SP500 index futures to pursue our prescribed objectives.

The structure of this paper is as follows. Section B reviews the Black OPM, the Black-Scholes OPM, the underlying assumptions, and related empirical works concerning the viability and use of these models. Data and methodology are described in Section C. As a basis for explaining and forecasting the ISD, Section D examines some of the distributional aspects of individual ISD's.

Section E attempts to fit the ISD data with autoregressive moving average (ARMA) structures in order to describe their time series nature, as well as forecast future values. As an additional tool for understanding the differences between individual ISD series and their non-constancy over time, separate cross-sectional time series regression models are developed for each of 1983 and 1984 data sets in Section F. ISD forecasts from the ARMA models and regression models are then compared with three naive methods and subsequently evaluated for accuracy.

To discern which ISD estimate has the greatest monetary value from a practitioner's point of view, in section G, hedging strategies between mispriced options and the underlying futures are devised and executed for predesignated holdout periods. Finally, in section H, the implications of our results are expressed from both academician and practitioner dominions of concern.

B. Review of Previous Research

The amount of research that has been conducted in relation to option pricing is substantial. This section seeks to briefly survey the major studies which form the impetus for our research.

Although this study indirectly involves the Black-Scholes model (1973), the extensive coverage it receives in the literature would make a review of its elements somewhat redundant. However, since we are studying futures call options, and Black's (1976b) model for pricing and deriving these contingent claims is used, we shall present his model here:

$$C_t^F = e^{-r\tau}[F_t N(d_1) - EN(d_2)] \tag{1}$$

$$d_1 = [\ln(F_t/E) + (\sigma_f^2/2)\tau]/\sigma_f\sqrt{\tau}$$

$$d_2 = d_1 - \sigma_f\sqrt{\tau}$$

where C_t^F is the model price for a futures call option at time t, F_t is the underlying futures price at time t, E is the exercise price of the call option, τ is the option's remaining time to maturity in terms of a year, r is the continuous risk-free rate annualized, σ_F^2 is the instantaneous variance of returns of the underlying futures contract over the remaining life of the option, and $N(\cdot)$ is the cumulative normal density function.

The differences in valuing an option on a futures contract versus an option on a stock can be seen by contrasting Black's model with the B-S model. Both models' strengths rely on the initial establishment of the riskless hedge portfolio between the option, its underlying asset and some riskless security. However, Black assumes that there are no up-front costs for entering into a futures contract as would occur if one was buying a stock. Thus in the Black model's derivation the interest term in the d_1 and d_2 components drops out. Additionally, Black implicitly assumes that the futures price series follows a sub-martingale, hence the futures price is an unbiased estimation of the contract's maturity price. It then follows through the derivation that the left hand portion of equation (1) ends up being discounted back to the present at the same rate as the exercise price (with the additional assumption that the option and futures contracts mature on the same date).

While these two pricing models are by no means identical, the general uniformity of their assumption and derivations will allow us to concurrently draw direct implications for the Black model and related but indirect consequences for the B-S model. In the context of this paper, we are concerned primarily with the assumptions that the instantaneous variance rate is proportionally constant over time. However, this study shall indirectly examine the model assumptions which assure a frictionless, liquid market to allow the costless formation and continuous adjustment of riskless hedge portfolios. The costs associated with low levels of volume are significant in option trading pits, particularly for deep-in and out-of-the money options and those with a long time to maturity. Evidence of such costs is exhibited in the wider range of bid-ask prices for these options.

While such a market reality might have some correlation with the B-S model's poorer pricing performance for such options, a growing array of evidence is emerging which points to the observed nonconstancy of the volatility parameter as a probable source of misspecification bias. Moreover, the proportionally constant variance and frictionless markets together imply that an adequately liquid level of trading volume exists for each option on a particular security. Generally, this implicit condition does not hold in the market place. As a result of these constraining assumptions, the individual risk-return characteristics between options differing by exercise price and/or maturity date, along with the particular market climate at hand may not be sufficiently expressed in the Black or B-S OPM framework.

Empirical tests of the "accuracy" of Black's model are not well founded in the literature, thus we turn to the abundance of research that has been generated off of the B-S model.

Although it is well confirmed that the B-S model exhibits biases in its pricing of deep-in and out-of-the-money options and those with a very short or very long term to maturity, the direction of bias is not reported consistently across studies. Black (1976a) found that the B-S model systematically over-priced options which were deep-in-the-money and underpriced those being deep-out-of-the-money. However, MacBeth and Merville (1979) reported an exactly opposite type of systematic bias. To make matters even more imprecise, Merton (1976) notes that practitioners often claim that the B-S model underprices both deep-in and out-of-the-money options. In regards to time to maturity, it is generally maintained that the B-S model underprices short-maturity and overprices long-maturity options. But again, the evidence contains discrepancies, particularly when the bias relative to both exercise price and maturity are considered. All these authors conclude that, to some degree, the pricing bias is related to the volatility parameter which is typically observed not to be proportionally constant over time as specified.

Several more generalized models have been proposed to overcome the B-S restriction on the volatility parameter. Merton (1976) derived a model based on a jump-diffusion process for the underlying security, that allows for discontinuous jumps in price due to unexpected information flows, for instance. An earlier formation by Cox (1975) called the constant elasticity of variance model, incorporates an observed market phenomenon that the underlying asset variance tends to fall as the asset price increases (and vice versa). Geske (1977) has derived a compound-option formula which considers the firm's equity to be an option underlying the exchange traded option. An interesting feature of Geske's model is that in incorporating the effects of a firm's leverage on its option the model allows for a nonconstant variance rate. Jarrow and Rudd (1982) focus on the potential effects from distributional misspecification of the underlying return-generating process. Thus, their model takes into account pricing biases which might arise due to differences between the second, third and fourth moments of the assumed and "true" distributions. Although tests of these models are far from conclusive, the general impression from the literature is that these models explain the B-S pricing biases better intuitively than they do empirically. However, extensive testing and use of these models is somewhat restricted due to the difficulty of accurately estimating their additional input variables.

Numerous studies have considered the biasing effects of dividend payments on the underlying stock which can invoke an early exercise value. In a recent study, Geske and Roll (1984) found that the American option variant of the B-S formula can only partially explain the bias associated with the B-S model in the theoretical value of an option.

Another route of model enhancement has gone the way of attempting to improve the estimation of the volatility term required by the B-S (and Black) models. Since Latane and Rendleman's (1976) development of the ISD concept, numerous researchers have studied different weighting schemes in calculating the ISD as well as its uses and pricing implications. The majority of studies, including Schmalansee and Trippi (1978) and Chiras and Manaster (1978) devise weighting schemes which aim at deriving a single weighted ISD from among all the individual ISD's to input into the B-S model. Whaley (1983) and Sears and Park (1984) utilized an OLS regression procedure to weight and segregate ISD's by maturity date. The major finding of Sears and Park's study, which like our study used option on stock index futures data, was a "time-to-maturity" effect in the pattern of the weighted ISD's over time. These authors interpreted their findings in light of Merton's (1973) OPM with stochastic interest rate, which implies that a portion of the ISD's composition is the diminishing instantaneous variance of the riskless security.

All the studies involving ISD estimation point out to one degree or another that for any day, the individual ISD's for all the options on a particular asset (stock or futures contract) will all be different, and will change over time. Yet as MacBeth and Merville aptly note, different exercise prices should not imply differing ISD's since the ISD pertains to the underlying asset itself and not the exercise price. In what might be considered a preliminary basis for this study, MacBeth and Merville relate systematic pricing differences between market and B-S option prices to the systematic differences that occur among individual ISD's relative to exercise price and time to maturity.

Another rather foreshadowing study conducted by Brenner and Galai (1981) not only found significant divergence between the daily individual ISD's and some time series average ISD, but that the distributions of the average ISD's were not invariant over time. Finally, Rubenstein (1981) uses individual ISD's to test five alternative option pricing models versus the B-S formulation, and attempts to explain observed pricing biases. In particular, Rubenstein noted that the direction of pricing bias changes over time which could be the influence of not only a time-varying volatility term, but also stochastic interest rates and a changing stock market climate.

C. Data and Methodology

Our data for the study of individual option ISD's included only the use of call options on the SP500 futures which are traded at the Chicago Mercantile Exchange (CME). We employed daily data gathered from the Wall Street Journal covering two periods of time; January 28 to June 30, 1983, and from February 29 to June 27, 1984. Studying two different time periods will allow us to discern some notion of ISD characteristics and movements over time as well as the effects of different market climates (1983 data were drawn from a "bullish" context, while the 1984 data came from more of a sideways, perhaps even "bearish" market). Interest rates on U.S. Treasury Bills (T-bills) were also gathered from the same source, updated daily, and computed as the converted equivalent bond yield from an average of the bid and ask discount rate for the T-bill, also having a maturity closest to the date of the option.

The calculations of daily ISD's for each quoted SP500 futures option were achieved using Whaley's method. Our application of this method differs from Whaley's in an important way, however. We choose to study individual option ISD's, broken down by maturity and exercise price. These ISD's are obtained by first choosing an initial estimate, σ_0, and then using equation (3) to iterate towards the correct value as follows:

$$C_j - C_j(\sigma_0) = (\sigma_1 - \sigma_0) \frac{\partial C_j}{\partial \sigma}\Big|\sigma_0 \qquad (3)$$

where

C_j = market price of call option j;

$C_j(\sigma_0)$ = theoretical price of call option j given $\sigma = \sigma_0$

σ_0 = initialized estimate of the ISD

σ_1 = estimate of the ISD from iteration

$\frac{\partial C}{\partial \sigma}|\sigma_0$ = partial derivative of the market price with respect to the standard deviation evaluated at σ_0

In the context of the Black OPM, the partial with respect to the standard deviation can be expressed explicitly as:

$$\frac{\partial C}{\partial \sigma} = e^{-r\tau} E(2\pi)^{-\frac{1}{2}} e^{-d_2^2/2} \sqrt{\tau} \tag{4}$$

The iteration proceeds by reinitializing σ_0 to equal σ_1 at each successive stage until an acceptable tolerance level is attained. We use the criterion

$$\left|\frac{\sigma_1 - \sigma_0}{\sigma_0}\right| < .0001$$

It should be emphasized that a unique feature of our approach is our determination of "maturity-" and "exercise price-specific" ISD's. We have disaggregated the option contract data in hopes of extracting information from ISD's that might otherwise be lost in conglomeration. The rationale for such a departure is as follows.

Although our discussions with various commodity brokerage houses indicated that most used some type of weighting scheme for ISD's, other talks with traders indicated a preference to consider each option as an individual (derivative) asset. So by utilizing a conglomerate or maturity-specific ISD, those options which are not near-the-money or moderately close to maturity, will appear to be mispriced. Yet, as other studies have shown, the B-S and Black models do not price far-in or far-out-of-the-money, or longer term to maturity options nearly as well as they do the nearer term, near-the-money call options. As indicated earlier, the assumptions underlying these models do not allow for market imperfections such as low liquidity and other various market idiosyncracies. Thus, the use of weighted ISD's would seem to impute a certain degree of "homogenization" into such options. More specifically, the pricing model used in conjunction with some weighted ISD would not account for such influences as differences in the level or consistency of volume on the option's actual trading price. Accordingly, all of our analysis and interpretation is focused on the ISD's of maturity- and exercise price-specific SP500 futures options.

D. Distributional Qualities of ISD's

The difficulty in discerning the correct value for the volatility parameter in the option pricing model is due to its fluctuation over time, thus complicating the estimate of its future value. Therefore, since an accurate estimate of this variable is so essential for correctly pricing an option, it would seem that time-series and cross-sectional analysis of this variable would be as important as the conventional study of security price movements. Moreover, by examining individual ISD's over time as well as within different time sets, the unique relationships between the underlying stochastic process and the pricing influences of differing exercise prices, maturity dates and market sentiment (and indirectly, volume), might be revealed in a way that could be modeled more efficiently. This section will examine the distributional qualities of ISD's as a prelude to our more quantitatively powerful ARMA and cross-sectional time series regression modeling in the next sections.

A summary of individual ISD distributional statistics for SP500 futures call options from '83 and '84 appears in Table 1. The most observable feature from this table is the significantly different mean values of ISD's that occur for different exercise prices. Looking at the means for the '83 ISD's, the average size and variability of the ISD appear to be inversely related to the size of the exercise price. Such a relationship could be related to volume in that deep-in-the-money options are traded less due to their larger relative cost, as well as their greater risk of being exercised (to those who sell them). Moreover, we might posit that a higher level of trading volume would be associated with a tighter bid-ask price spread in the pits, and consequently, a lower volatility of prices as might be reflected in the ISD's. More conclusive evidence of the relationship of ISD's to exercise price will arise from the results of the cross-sectional time series regression in the next section.

Comparing the mean ISD's across time periods, it is quite evident that the 1984 ISD's are significantly smaller. This decline in the size of the average ISD could very well be related to the change in

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Insert Table 1 about here
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market climates between 1983 and 1984. Also noticeable is an alteration in the inverse relationship between the mean size of the ISD and exercise price, as observed for the '83 data. Although the June options still tend to follow this same pattern to a smaller degree, the September options display ISD's which decline from the high and low exercise prices towards a low value for the at-the-money option. Such a pattern for these intermediate term options might be reflective of a larger concentration of trading volume for the at-the-money option, perhaps due to indistinct market sentiment. We can also tentatively identify a time-to-maturity effect as was observed by Park and Sears (1984), with the

September options possessing higher mean ISD's than those maturing in June. Once again, stronger support for this effect on the ISD's will be displayed in the regression results.

While it is not empirically conceivable at this point to draw conclusions concerning the size and variability of the ISD and their relationship to trading volume and market sentiment, Tables 2a and 2b do provide some intuitive fortitude to these conjectures. These tables show typical price and volume data on a single day for the SP500 futures options in '83 and '84. As can be easily seen, the concentration of volume correlates very well (negatively) with the size and variability of the ISD's. Moreover, while the highly bullish '83 market shows higher volume at higher exercise prices, the rather unclear, somewhat bearish market in the spring of '84 indicates significantly more trading concentration around lower exercise prices (160-165).

Insert Tables 2a and 2b about here

The other statistical measures listed in Table 1 are the relative skewness and relative kurtosis of the ISD series, along with the studentized range. Skewness measures lopsidedness in the distribution and might be considered indicative of a series of large outliers at some point in the time series of the ISD's. Kurtosis measures the peakedness of the distribution relative to the normal and has been found to affect the stability of variance (Lee and Wu, 1984). The studentized range gives an overall indication as to whether the measured degrees of skewness and kurtosis have significantly deviated from the levels implied by a normality assumption for the ISD series.

Although we prefer to postpone any interpretation of the effects of skewness and kurtosis on the ISD series until the regression results, a few general observations are warranted at this point. For instance, the '83 statistics suggest that the individual ISD series approximated a normal distribution. However, the '84-related statistics present a very different view of this, certainly challenging any assumptions concerning normality. Using significance tests on the results of Table 1 in accordance with Snedecor and Cochran (1967), the 1984 skewness and kurtosis measures indicate a higher proportion of statistical significance, although not clearly more than that which occurred among the 1983 measures. At this point we can only surmise that the contrast of statistics would be related primarily to effective changes in the market climate.

As a final point to this brief examination of the ISD skewness and kurtosis, we note the statistics for the 1983 June 150 contract. The relative size of this contract's skewness and kurtosis measures reflect the high degree of instability that its ISD exhibited during the last ten days of the contract's life. Such instability is consistent across contracts. However, we have allowed these distortions to remain in the computed skewness and kurtosis measures only for this particular contract to emphasize how a few large outliers can magnify the size of these statistics. Thus, while still of interest, any skewness and kurtosis measures must be calculated and interpreted with caution.

E. Time Series Analysis of Implied Standard Deviations

The time series model building techniques we use are stemmed in large part from Box and Jenkins (1970) who proposed a three stage iterative procedure of identifying, estimating, and checking models describing particular generating processes. These models are of the form

$$x_t - \Phi_1 x_{t-1} - \ldots - \Phi_p x_{t-p} = \varepsilon_t - \theta_1 \varepsilon_{t-1} - \ldots - \theta_q \varepsilon_{t-q} \tag{5}$$

where x_t is an observation from a covariance stationary series meaning that

$$\lambda_\tau = \mathrm{cov}(x_t x_{t-\tau}) \tag{6}$$

is independent of t for all τ. If stationarity conditions are not satisfied, they can typically be induced by redefining the x_t's to be the first differences between successive observations. The Φ and θ terms represent the autoregressive (AR) and moving average (MA) coefficients and ε_t is white noise. A large body of evidence is accumulating which supports the Box-Jenkins methodology, especially in cases of single series with moderate to large numbers of sample observations.

If any stage of the iterative process falls short of being incontrovertibly clear, it is the initial one. The theoretical relationships between autoregressive moving average structures and concomitant autocorrelations and partial autocorrelations are often useful in selecting a model that adequately describes a sample data set, yet detecting these theoretical patterns in practice can rightly be considered more of an art than a science.

A recently developed technique motivated by Hannan and Rissanen (1982) seems to provide a good practical basis for model selection. The process involves two stages of computation. The purpose of the first stage is to obtain estimates of the innovation errors of the ARMA model. This is accomplished by running successively higher order autoregressive models and using the AIC of Akaike (1969) to determine the optimal order from among them. The innovation errors are estimated by

$$\hat{\varepsilon}_t = x_t - \hat{\Phi}_1 x_{t-1} - \cdot \cdot \cdot - \hat{\Phi}_k x_{t-k} \tag{7}$$

where k is the optimal autoregressive order suggested by the AIC. The second stage involves fitting all different combinations of ARMA(p,q) models where, instead of using full maximum likelihood estimation, the innovation errors estimated in stage one are used as the regressors upon which the moving average parameter estimates are based. This allows us to use least squares, thereby saving copious quantities of computer dollars. These different ARMA(p,q) models ($p \leq 5$, $q \leq 5$ have been used) are then compared using the BIC of Akaike (1977) and Schwarz (1978) and the appropriate model is chosen on that basis. This procedure comes with no guarantees of consistently being able to determine "correct" model structures, yet it has been very valuable, when used in conjunction with sample autocorrelations and partial autocorrelations, in providing good first guesses.

Once the values of p and q are chosen in the initial stage, the parameters are estimated. A simulation study conducted by Ansley and

Newbold (1980) has found that exact maximum likelihood estimation outperforms least squares when the series are of moderate size and moving average terms are involved. An approximation to the full maximum likelihood function has been derived by Hillmer and Tiao (1979) and is the basis for the estimation program put forth by Tiao, et al. (1979) that we use.

Our fitted models are then subject to a series of diagnostic checks to ensure that the initially specified structures are indeed adequate. These checks may be viewed as either tests against alternative specifications involving additional AR or MA terms or tests based on the residual autocorrelations from the fitted models. See Newbold (1984) for details of these test statistics. If these tests fail to support the initially specified ARMA structures, we would begin again at the model identification stage, with more elaborate ARMA structures to consider.

We examined the sample autocorrelations of each of the ISD series and they tended to die out quickly enough over successive lags that the stationarity assumption appears to be satisfied without first differencing. The series were then modelled in the manner just described. After satisfactory models were obtained, it remained to find the best use for the information afforded us by the parameter estimates. Certainly the more predictable future ISD's are the more profitable ones hedged trading strategies become. In order to test the accuracy of the ARMA forecasts, we calculated the mean squared forecast errors of one-day-ahead forecasts for holdout periods of different lengths and compared them with the mean squared forecast errors obtained by using previous period

ISD's, 5-day moving averages, and 5-day exponential moving averages. We also compare these forecasts with those derived from the regression model developed in the next section. The holdout periods range from five to 20 days to indicate how different predictors perform over different forecast periods. Some of the data sets were not tested over the longer 10 or 20 day holdout periods because there were too few observations in the series to do so.

As Tables 3 and 4 indicate, there is no one clearly superior forecasting method. This may be a bit perplexing since three of the alternatives to the ARMA forecasts that we consider are actually very specific cases of ARMA models themselves where the structural and quantitative relationships of the time series with their forecasts are predetermined.[1] In contrast, the efficiently specified and estimated ARMA models are chosen on the basis of the data and would therefore be expected to better project particular data generating processes into the future. The best explanation for the poorer than expected ARMA performance is that our forecast periods, at times, exhibit structural change or even a sudden trend in ISD's. Non-stationarity (trend behavior) was not observed in the estimation period and hence the ARMA coefficients that were derived for that time could not predict any stationarity in the holdout period. Some of the other, "naive" forecastors are more responsive to these sudden movements. They do not have as much of a tendency to pull their forecasts toward middle ground as ARMA models do. For example, the most naive forecast ($ISD_{t+1} = ISD_t$) will be better than some ordinarily superior ARMA forecast (e.g., $ISD_{t+1} = a + b\ ISD_t$) in times of unanticipated non-stationarity.

Insert Tables 3 and 4 about here

To tie together some of the results we have identified thus far, we point out an interesting relationship that arises from an examination of the MSFE's from the ARMA forecasts (Tables 3 and 4) and the standard deviations of the ISD series (Table 1). While not precise, it is apparent that the accuracy of the ARMA forecasts correlates with the variability of the ISD series. Hence, the more volatile the ISD, the more difficult it is to forecast.

We realize that this observation might be obvious yet we suggest a further implication. For instance, one could find such information valuable in deciding which particular options might be mispriced based on the use of the OPM and some forecast of the ISD. For instance, if one calculates that both the September 155 and the September 165 call options are underpriced, with a significantly lower standard deviation of the ISD for the 165 call, one might feel justified in putting more merit behind the latter being mispriced.

F. Regression Model

A significant amount of information has been shown to exist in a time series of ISD's. The ARMA models used to describe the generating processes of the ISD series we have examined are all clearly preferred to random walk or "white noise" alternatives. These ARMA models do not give the final word on the subject of ISD forecasting, however. There are several cross-contract effects that may exist which, if isolated properly, will provide further predictive power. In addition, the ARMA models have not fully captured several time effects. To learn more about these different influences, we formulated a large cross-section time series predictive regression model and ran it on the 1983 data and again on the 1984 data. The regression model is

$$y_{it} = \beta_0 + \beta_1 x_{1it-1} + \beta_2 x_{2it-1} + \ldots + \beta_{14} x_{14it-1} + \varepsilon_{it} \tag{8}$$

where

y_{it} = ISD of the i^{th} contract at time t

x_{1it-1} = optimal time series predictor of i^{th} contract for time t (based on information known at time t-1)

x_{2it-1} = time to maturity

x_{3it-1} = (futures price - exercise price)/(exercise price) if > 0, 0 if not

x_{4it-1} = (exercise price - futures price)/(exercise price) if > 0, 0 if not

x_{5it-1} = standard deviation of the ISD based on previous 5 observations

x_{6it-1} = standard deviation of the ISD based on previous 20 observations

x_{7it-1} = skewness of ISD distribution over the previous 20 observations

x_{8it-1} = kurtosis of ISD distribution over the previous 20 observations

x_{9it-1} = standard deviation of continuous futures returns using previous 5 observations

x_{10it-1} = standard deviation of continuous futures returns using previous 20 observations

x_{11it-1} = dummy variable; 1 if Tuesday, 0 if not

x_{12it-1} = dummy variable; 1 if Wednesday, 0 if not

x_{13it-1} = dummy variable; 1 if Thursday, 0 if not

x_{14it-1} = dummy variable; 1 if Friday, 0 if not

The optimal time series predictors are included in an attempt to educe information contained in the past. This variable is likely to capture large portions of the expected cross-contract effects since the market influences pertaining to a particular contract today are not likely to have changed considerably since the prior trading day. The resulting ISD's tend to evolve over time with a strong sense of heritage.

The time-to-maturity variable was included because, as was indicated by Sears and Park (1984), there tends to be a certain point close to maturity where the ISD's begin to decrease. Of course, any general downward trend in ISD's would be partially accounted for by the ARMA predictors, but it may still be the case that there is a partial influence that time-to-maturity exhibits.

The third and fourth independent variables have been included to see if deep-in-the-money options and far-out-of-the-money options tend toward higher or lower than expected ISD's. Previous studies have had conflicting answers to this important question, according to Jarrow and Rudd (1983).

The next two independent variables are included to determine whether or not the standard deviations of the ISD's have any positive or negative effect on the ISD's themselves. The third and fourth moments of the distribution of 20 previous ISD's were also included in the regression equation to see what, if any, influence they have in determining current ISD's.

The two measures of the standard deviations of continuous futures returns are of great interest as regressors since these have traditionally been approximations of the variable used in the B-S model to determine the theoretical option price. One would hope to find a strong relationship between the two volatility measures; the one based on historical deviations and the one implied by the observed option price. It may be the case, however, that the implied standard deviation encompasses more than just the expected future standard deviation of the underlying asset's return. All B-S model misspecifications are represented in the ISD term, which may amount to quite a large distortion. A low correlation between these historical and implied variables would indicate either that the model misspecifications manifesting themselves in the ISD terms are significant or that the historical standard deviation measure is a poor proxy for the expected future standard deviation, or both.

The final four explanatory variables are day-of-the-week dummies which are intended to see if certain days give rise to higher ISD's than others. For example, certain economic announcements are regularly made on particular days of the week and this may have a day-of-the-week effect on ISD's. Note that we include only four dummy variables to describe the five days of the week in order to avoid perfect multicollinearity with the constant term.

Determining the correct form of a model using pooled cross-section and time series data is an important, though often troublesome task. The difficulty arises because the error term may consist of time-series-related disturbances, cross-section disturbances, or both. The regression equation we have defined with the optimal time series predictor in the design matrix, gives us confidence that any possibility of autocorrelation in the time series disturbances has been mitigated. There remains the chance that contemporaneous correlation of disturbances across contracts exists. However, we feel a majority of this cross-contract relationship will be accounted for by the in-the-money and out-of-the-money variables. These are akin to the cross-section dummy variables of the so-called covariance model. The only real difference between the two sets of variables is that the estimated dummy coefficients can take on any values whereas our in-the-money and out-of-the-money variables implicitly assume that a linear relationship exists between a contract's exercise price above the underlying asset price and its ISD and between a contract's exercise price below the underlying asset price and its ISD. If this posited linear relationship does exist, then cross-contract disturbances should not be highly correlated. The proper estimation procedure, given our assumed error structure:

$$\varepsilon_{it} \sim (0, \sigma_\varepsilon I_m) \quad i=1,2,\ldots,N \quad t=1,2,\ldots,T_i \quad m = \sum^{N} T_i$$

is ordinary least squares. Of course, we will need to check the residuals for possible violations of our assumptions before placing confidence in the sample results.

The regression results are given in Table 5. The optimal time series predictors were significant, which should come as no surprise--ISD's depend on past ISD's. The fact that other regressors were found to be significant indicates that not all of the variation in ISD's is explained by the past, though. Time-to-maturity has its predicted positive effect. The closer an option is to expiration, the lower the ISD. Both the in-the-money and out-of-the-money effects are significantly positive. This result supports the findings of Merton (1976) who showed that large deviations from the strike price tend to bias the B-S theoretical price downward. It is logical to expect the ISD's of the more thinly traded in-the-money and out-of-the-money contracts to be higher because the writer of these calls runs a greater risk of

Insert Table 5 about here

being stuck in his position. To compensate for the added risk, the seller would gross up the price thus making the ISD's for these contracts higher. Individual contract volume data would surely serve to verify this assertion. (See, as an example of volume by contract, Tables 2a and 2b).

The coefficients on the standard deviation of the ISD variables give mixed signals, if they can be considered signals at all. This result is not terribly disappointing since there were no strong a priori notions of the effect of ISD variability on the ISD level, anyway.

The skewness term has a negative effect in the 1984 model and the kurtosis term has a positive effect in the 1983 model. These results are difficult to interpret, especially given that the effects are not consistently significant over the two sample periods. A conceivable cause of this muddled picture is the unaccountable temporal ordering of the more influential observations shaping the distribution. For example, if several outliers have been observed to skew a distribution, the predictive signal this information might provide would depend on whether these outliers were observed recently or in the more distant past. Perhaps what can be said about the negative relationship between skewness and predicted ISD's is that there is a tendency to discount the influence of the outliers bringing about the skewness, especially when the outliers comprise the optimal time series predictor. Discounting the influence of the skew results in predictions closer to the mean of the ISD distribution--a reasonable direction to go if you believe such a market correction mechanism exists.

The coefficients on the standard deviation of continuous futures returns variables switch signs going from the 5-day measure to the 20-day measure. This is likely an artifact of collinearity between the two measures rather than any genuine partial effect on ISD. Interestingly, separate regressions of the ISD on the 5-day return standard deviation alone and on the 20-day return standard deviation alone indicate very low correlations. These particular regression results serve to underscore the nature of the <u>implied</u> standard deviation. The ISD not only reflects the market's assessment of the future instantaneous standard deviation but also the market's deviation from the functional specification of the OPM. Therefore, its relationship to historical standard deviations may not be strong.

The day-of-the-week dummies indicate a small Friday effect where the ISD's are slightly higher. This may be related to the fact that certain economic announcements are made on Thursdays (e.g., money supply figures) that will alter the market perception of asset price volatility. The Friday effect might also be related to option market inactivity the day before the weekend. Further study may illuminate this apparent day-of-the-week effect, explaining why Friday's market may be out of line with that of other days.

A visual inspection of the residual terms has confirmed the underlying assumptions mentioned earlier concerning the error structure of this model. There is no apparent serial correlation or contemporaneous correlation so our seemingly insouciant attitude toward more complicated GLS model structures is justifiable.

Whether the estimated models change significantly over time is an important question. The parameter estimates obtained for this cross-section time series model did change significantly from the 1983 sample period to the 1984 sample period. A Chow test statistic indicating structural change was obtained for the 1983 and 1984 regressions and was calculated to be 2.30.[2] This value exceeds the table value of 2.04 for an F random variable at the 99 percent level. It would therefore be wise for the practitioner to update parameter estimates periodically. Perhaps the use of switching regressions would prove worthwhile in determining the points at which the structural changes occur. The switch points could very likely correspond to changes in market sentiment. The observed structural change going from our 1983 sample period (bullish market) to our 1984 sample period (mixed, more downward looking market) may be evidence of a prominent market sentiment effect, though no explicit tie is necessarily implied.

G. Trading Rules--Comparative Results & Implications [Ex-Post Test]

In the previous section, our various estimates (ARMA and regression) for the "true-future" underlying ISD were compared to a number of naive methods and evaluated through the conventional measure of forecast accuracy, the MSFE. Now we propose to test the practical (monetary) value of our ISD estimates versus more naive methods, to determine which might be superior from a trader's point of concern. In addition, we hope that these results will further support the theoretical and practical superiority of using individual ISD estimates versus some weighted-ISD measure.

Our discussions with traders of SP500 futures options along with various segments of the brokerage industry yielded a general view of how ISD's are utilized in practice. The common methods that emerged were to look at yesterday's weighted ISD (though some sources indicated that they looked at individual ISD's), or some type of moving average scheme of past ISD's as being the true-future underlying volatility to use in the Black OPM. Changes in the option's price due to deviations in the current ISD from the estimates of the true-future ISD are thus deemed as mispricings by the market. Our trading rule tests utilized six different estimates for the volatility parameter which were:

(1) a 5-day exponentially-weighted average of the ISD

(2) a 5-day equally-weighted moving average of the ISD

(3) 1-day ahead ARMA forecasts of the ISD

(4) a 1-day lag of the actual ISD for the option

(5) 1-day ahead regression forecasts of the ISD

(6) a simple-constant mean value for the ISD, representative of the near-the-money, nearer term (high volume) option.

The trading rule that we will use is simply to buy underpriced and sell overpriced options, while taking an opposite position in the underlying futures contract according to the hedge ratio (computed using the estimated ISD). Mispricings will be identified by comparing the closing market price for an option with the Black OPM price using one of the seven ISD estimates. The holdout periods for each option are the same as those from the last section, ranging from eight to twenty actual trading days. In each transaction ten options were bought or sold (and 10x the hedge ratio of futures were sold or bought, respectively), in order to magnify the mispricings as might be seen from the eyes of a trader, as well as to make the correct hedge ratio more realistically attainable. Positions are closed out (on the basis of closing prices) once the mispricing diminishes to a predetermined minimum level. If the mispricing has reversed itself and is of a great enough significance, the position is assumed to be reversed on the same day that it is closed out. While a number of trading rules were tested, the one we used for the results of this paper is summarized in Table 6.

Insert Table 6 about here

In order to ascribe as much realism as possible to these tests, we considered the following market trading costs. Commission costs per transaction of \$20 were determined by using the rates publicly offered by a discount futures broker. These costs were calculated as follows:

$$\begin{array}{ll} \sum^{n}(\$20 \times 10) & : \text{ cost of option position} \\ \underline{+\sum^{n}(\$20 \times 10 \times \text{hedge ratio})} & \underline{: \text{ cost of future position}} \\ \text{Total Commission Costs} & \end{array}$$

n = the total number of times a position was opened.

Although a portion of the margin required of a trader enter into a futures position can be put up in the form of interestearning T-bills, a substantial portion required for maintaining the margin account by the clearinghouse must be strictly in cash (even for a hedge or spread position). Consequently, there is a real interest cost involved, for which we will further reduce gross trading income:

$$\text{Margin Interest Costs} = \sum^{n}(\text{RMM} \times NF_i \times i \times \tau_i)$$

where: RMM = required maintenance margin; NF_i = number of futures contracts entered into; i = interest cost (assumed to be 10.0%); τ_i = length of futures position in annual terms; and n = total number of times a futures position was entered.

Some other very real and significant market costs are those associated with liquidity and timing. Options which are deep-in-the-money for instance, are not heavily traded and therefore present additional costs for actually getting into or out of a position. Since the last reported price for the option may not have occurred at the same time as that for the underlying future, there always arises a problem with using closing prices. Furthermore, there is little assurance that one could buy or sell these contracts and expect to receive the closing prices reported in the paper when the market reopens the next morning. To approximate such market costs we first penalize ourselves each time we enter and exit a futures position by "one tick" (equal to a price change of .05 = $25):

$$\text{Futures Liquidity Costs} = \sum^{n} (\$50 \times NF_i)$$

where: $50 = (2 x .05 x $500), the market value of two price ticks; NF_i = number of futures entered in any one trade; and n = number of times a futures position was entered.

More severe liquidity (and timing) costs were calculated and deducted for each option transaction:

$$\text{Option Liquidity Costs} = \sum^{n} [\$5000 \times (NEPA \times .1 + NMMO \times .1)]$$

where: $5000 = 10, (number of options bought or sold) x $500 (the market value multiplier for the option premium); NEPA = number of exercise prices (in increments of $5) away from being at-the-money; NMMO = number of maturity months out; and 0.10 = correspondingly liquidity cost, (two price ticks).

The results of our tests are summarized in Tables 7 through 9. Tables 7a and 7b summarize the cumulative trading results for all options studied in a particular year, by which ISD estimate was used in the calculation of the model price. For both years, the forecast from the regression model proved to be superior overall. This outcome lends support to the advantage of considering the direct effects of differing exercise prices when trying to predict the ISD for an option.

The regression model utilized some of the insights of time series analysis as would be impounded in the optimal time series predictors. Also, this cross-sectional time series regression model took into account the historical 20-day standard deviation of the continuous return for the underlying futures contract, the short-term variability and skewness and kurtosis of the ISD (indirect effects of exercise price and perhaps market sentiment), the time-to-maturity, and the day of the week.

We might also note that nearly any method would have made money in '83, while '84 was certainly a less volatile market with little trend, and consequently was more difficult to profit in. The ARMA forecasts did quite well overall as did the straight moving average (surprisingly), while the exponential moving average performed particularly well in 1983.

Though certainly not conclusive or even completely realistic, these results do point to the additional and useful information which is impounded in the individual ISD's which would have been lost by aggregating them. That such information is available to potentially profit from, suggests that either the market is inefficient and/or the Black OPM is misspecified (and perhaps correspondingly the B-S OPM).

Insert Tables 7, 8 and 9 about here

H. Summary & Concluding Remarks

The purpose of this paper has been to improve the interpretation and forecasting of individual implied standard deviations for call options on SP500 futures. By empirically explaining their composition through time series analysis and cross-sectional time series regression models, and profitably utilizing this information to identify mispriced options, we have demonstrated the disadvantages to using individual option ISD measures. More importantly, our results provide some evidence as to how the Black option pricing model (and relatedly the Black-Scholes model) might be misspecified, or jointly, how the market might be inefficient. Though the original model implicitly assumes a frictionless market and a constant volatility term, market realities along with past studies would not be able to substantiate these types of assumptions. Moreover, the indirect effects of volume as exhibited through differences in exercise price, time to maturity, and even market sentiment over time, are verified by these results as being of consequence for the "true" valuation process of an option. Yet, the Black OPM takes no explicit account of such market imperfections and subjectivities. The misspecified aspects of the model appear to get "dumped" into the ISD variable, empowering it to represent something more than just a measure of volatility for the underlying asset returns. This paper has found that the underlying generating process for the ISD is forecastable, therefore it is conceivable to base trading rules on the informational content of this variable alone.

FOOTNOTES

[1]The first, most naive predictor considered is the ISD from the previous period. It is simply an AR(1) process of the form $ISD_t = 0 + 1 \cdot ISD_{t-1}$. The 5-day moving average predictor can be expressed as an AR(5) model of the form $ISD_t = 0 + .2 \cdot ISD_{t-1} + .2 \cdot ISD_{t-2} + \ldots + .2 \cdot ISD_{t-5}$. The approximate 5-day exponential moving average we used is another special case of the general AR(5) model where the parameters are restricted as follows: $ISD_t = 0 + 16/31 \cdot ISD_{t-1} + 8/31 \cdot ISD_{t-2} + 4/31 \cdot ISD_{t-3} + 2/31 \cdot ISD_{t-4} + 1/31 \cdot ISD_{t-5}$.

[2]Chow test: $F_{q,n-k} = \dfrac{(e_*'e_* - e'e)/a}{e'e/(n-k)} = \dfrac{(.004370-.004075)/15}{.004075/(507-30)}$

$e_*'e_*$ is restricted SSE

$e'e$ is unrestricted SSE

a is number of restrictions

K is number of regression coefficients estimated in unrestricted regression

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TABLE 1

DISTRIBUTIONAL STATISTICS FOR INDIVIDUAL ISD'S

	Option Series	Mean	Std. Dev.	Coefficient of Variation	Skewness	Kurtosis	Studentized Range	Obs.
1983								
	#June 150	.20282	.04511	.222	4.860*	27.274	7.766*	95
	June 165	.15416	.01963	.127	-.205	.180	4.774	70
	Sept. 145	.19839	.02438	.123	.292	-.308	4.782	106
	Sept. 150	.18334	.02161	.118	-.284	.427	5.375	106
	Sept. 160	.17452	.01454	.083	-.484*	-.209	4.874	86
	Sept. 165	.16372	.01468	.090	-.451*	-.283	4.777	64
	Sept. 170	.15794	.01104	.070	.612*	-.299	3.999	44
	Dec. 165	.15789	.02087	.132	-.864*	.397	4.496	53
1984								
	June 150	.13168	.00781	.059	-.762*	1.435*	5.383**	52
	June 155	.12490	.00629	.050	.189	-.767*	4.071**	50
	June 160	.11944	.00630	.053	.116	-1.280*	3.465**	50
	June 165	.11865	.00536	.045	-.175	.787*	3.740**	47
	Sept. 160	.12938	.00452	.035	-.159	-.095	4.871	57
	Sept. 165	.12876	.00510	.040	.843*	1.501*	5.323**	62
	Sept. 170	.12910	.00570	.044	.384*	.047	4.426	62

#data contained large outliers which occurred as the option moved very near to expiration.

*indicates that the statistic is significant at the 95 percent level of confidence.

**indicates that the statistic is significant at the 95 percent level of significance.

Table 2a

THURSDAY, MAY 19, 1983 — S & P 500 OPTIONS

CALLS-	STRIKE	OPEN RANGE	HIGH	LOW	CLOSING RANGE	SETT. PRICE & PT. CHGE.	FUTURES SETT. PRICE	++IOM RISK FACTOR	EXCER-CISES	VOLUME (TRADES CLEARED)	OPEN INTEREST
Jne83	140.00	---	23.15B	---	23.00N	22.15 - 85	162.15	.986	---	---	4 unch
Jne83	145.00	---	18.40B	---	18.00N	17.15 - 85		.954	---	---	11 unch
Jne83	150.00	---	13.40B	12.30A	12.30A	12.30 - 80		.881	---	4	370 - 2
Jne83	155.00	8.70	9.05	7.40	7.70B	7.70 - 80		.758	---	12	893 - 8
Jne83	160.00	4.60	4.90	3.35	3.65@.55	3.70 - 75		.591	---	146	1673 + 94
Jne83	165.00	1.85	2.00	1.15	1.30	1.30 - 40		.412	---	374	1769 + 2
Jne83	170.00	0.60	0.65B	* 0.35	0.35	0.35 - 15		.254	---	506	641 - 204
Jne83	175.00	0.10	0.10	* 0.10	0.10	0.10 - 10		.138	---	25	202 + 25
Sep83	140.00	---	---	23.10A	23.10A	23.10 - 190	163.25	.879	---	---	2 unch
Sep83	145.00	---	---	18.50A	18.50A	18.50 - 150		.821	---	---	10 unch
Sep83	150.00	---	---	14.00A	14.00A	14.00 - 200		.750	---	---	12 unch
Sep83	155.00	11.50	11.80B	10.90A	10.90A	10.90 - 70		.670	---	1	62 unch
Sep83	160.00	7.95	8.25	7.75A	7.75A	7.75 - 105		.585	---	14	81 + 9
Sep83	165.00	5.80	5.80	5.00A	5.00A	5.00 - 80		.497	---	3	151 + 1
Sep83	170.00	3.65	3.65	* 3.10	3.20B	3.20 - 40		.413	---	23	220 + 22
Sep83	175.00	2.00	# 2.60B	* 1.75A	1.75A	1.75 - 75		.335	---	2	10 + 2
Dec83	165.00	---	---	6.80A	6.80A	8.80 - 50	164.65	.532	---	---	6 unch
Total S & P Calls:									---	1110	6137 - 59

S & P 500 STOCK INDEX-				
Jne	163.65@.50	163.80	161.65	162.05@.25
Sep	164.65@.75	164.90	162.90	163.20@.15
Dec	166.05	166.10B	164.40	164.65A
Mar84	---	167.20B	165.60A	165.85A
Total S&P 500 Stock Index:				
CASH MARKET:		163.61	161.98	

Table 2b

THURSDAY, MAY 17, 1984 — I.O.M. S & P 500 CALL OPTIONS — SIDE 2 of 1 PAGE

STRIKE	OPEN RANGE	HIGH	LOW	CLOSING RANGE	SETT. PRICE & PT. CHGE.	IOM RISK++ FACTOR	EXER-CISES	VOLUME (TRADES CLEARED)	OPEN INTEREST	---CONTRACT--- HIGH	---CONTRACT--- LOW
JNE84 (FUTURES SETT.--- 157.05)-											
150	7.90	7.90	7.50A	7.50A	7.30 - 140	.925	---	1	31 + 1	13.50B	6.55
155	4.10	4.10	3.25	3.30	3.30 - 120	.664	---	86	1352 + 6	9.60B	3.25
160	1.45	1.45	*0.95	1.05	1.00 - 55	.287	---	279	5660 + 101	5.90	0.95
165	0.35	0.35	*0.20	0.25	0.25 - 10	.065	---	379	5518 - 90	9.75B	0.20
170	0.05	0.05	0.05	0.05	0.05 unch	.007	---	57	1202 - 50	7.50	0.05
175	---	---	---	0.05N	0.05 unch	.000	---	---	385 + 1	4.40	0.05
180	---	---	---	0.002N	0.002 unch	.000	---	---	182 unch	2.40	0.05
185	---	---	---	0.002N	0.002 unch	.000	---	---	57 unch	4.00	0.002
					Total Jne84:		---	802	14387 - 31		
SEP84 (FUTURES SETT.--- 159.70)-											
155	---	---	*6.90A	8.15N	7.00 - 115	.685	---	---	22 unch	11.00B	6.90A
160	4.50	4.50	*4.20	4.40	4.25 - 90	.502	---	102	170 + 53	8.50	4.20
165	2.75	2.75	*2.40	2.45	2.30 - 70	.324	---	205	458 + 197	5.50B	2.40
170	---	---	*1.40A	1.40A	1.30 - 50	.183	---	10	76 + 10	5.20	1.40A
175	---	---	*0.80A	0.80A	0.70 - 20	.090	---	---	19 unch	6.00	0.80A
180	0.45	0.45	*0.40	0.40	0.30 - 15	.039	---	13	82 + 13	3.85	0.40
					Total Sep84:		---	330	827 - 273		
TOTAL S & P 500 CALL OPTIONS:							---	1132	15214 + 242		

Table 3

Mean Squared Forecast Errors for Different Predictors of ISD_t

		Forecasting method used				
Series	Holdout Period	Previous ISD	5-day moving avg.	5-day exponential moving avg.	ARMA	Regression
June 150 (83)	5	.205	377	.264	.198	.298
	10	.213	413	.268	.226	.335
June 165 (83)	5	.034	081	.046	.049	.046
	10	.085	067	.060	.092	.069
Sept 145 (83)	5	1.727	626	.901	1.039	.837
	10	.963	529	.592	.614	.463
	20	.541	428	.373	.509	.384
Sept 150 (83)	5	.733	669	.833	.818	.742
	10	.477	456	.524	.535	.495
	20	.700	589	.424	.440	.373
Sept 160 (83)	5	.119	075	.074	.203	.162
	10	.139	079	.087	.187	.163
	20	.101	130	.094	.128	.103
Sept 165 (83)	5	.056	200	.107	.017	.171
	10	.035	123	.063	.018	.073
Sept 170 (83)	5	.023	025	.016	.012	.009
Dec 165 (83)	5	1.255	770	.741	.945	.738
	10	.770	600	.545	.592	.456

(MSFE's given in the table are scaled by a factor of 10^4)

Table 4

Mean Squared Forecast Errors for Different Predictors of ISD_t

Series	Holdout Period	Previous ISD	5-day moving avg.	5-day exponential moving avg.	ARMA	Regression
		Forecasting method used				
June 150 (84)	5	.029	.036	.029	.024	.027
June 155 (84)	5	.048	.110	.066	.087	.147
June 160 (84)	5	.000	.001	.000	.001	.001
	10	.005	.008	.006	.006	.006
June 165 (84)	5	.041	.026	.026	.026	.026
	10	.063	.037	.033	.031	.031
Sept 160 (84)	5	.005	.015	.009	.021	.015
	10	.005	.018	.010	.023	.026
	20	.014	.029	.019	.031	.032
Sept 165 (84)	5	.007	.007	.006	.004	.010
	10	.011	.016	.013	.010	.016
	20	.010	.018	.013	.010	.011
Sept 170 (84)	5	.014	.010	.010	.015	.025
	10	.017	.014	.014	.015	.019
	20	.014	.011	.011	.012	.016

Table 5

Regression Results

Variable	1983	1984
Optimal time series predictor	.698 (8.35)***	.195 (2.94)***
Time to maturity	.00029 (1.36)	.00081 (7.58)***
Proportion in-the-money	.0351 (4.11)***	.0450 (6.89)***
Proportion out-of-the-money	.0105 (0.56)	.00778 (1.81)*
σ_{ISD}, 5 obs.	-.199 (-0.90)	.454 (2.47)***
σ_{ISD}, 20 obs.	-.502 (-1.51)	-.262 (-1.05)
Skewness	-.00025 (-0.43)	-.00029 (-2.06)**
Kurtosis	.00054 (1.87)*	-.00003 (-0.55)
σ_{return}, 5 obs.	-.214 (-1.74)*	.150 (4.46)***
σ_{return}, 20 obs.	.689 (2.89)***	-.246 (-2.20)**
Tuesday	.00037 (0.44)	-.00004 (-0.16)
Wednesday	.00071 (0.84)	.00008 (0.34)
Thursday	.00016 (0.19)	-.00013 (-0.57)
Friday	.00146 (1.74)*	.00027 (1.13)
R^2	.642	.679
Adj. R^2	.619	.661
S_e	.00397	.00119

(Figures in parentheses are t-statistics).
*Indicates significance at 90 percent level.
**Indicates significance at 95 percent level.
***Indicates significance at 99 percent level.

Table 6

Summary of Trading Rule Used to Form Neutral Hedge Portfolios

Classification of the Option	Min. Amt. of Mispricing Required to Open a Position	Max. Amt. of Mispricing Allowed Before Closing a Position
(Futures price - exercise price) > $10	.35	.05
(Futures price - exercise price) > $5 < $10	.30	.05
(Futures price - exercise price) > $1 < $5	.25	.025
(Futures price - exercise price) > $.2 < $1	.20	.025

Table 7

Cumulative Survey of Trading Results

a) 1983

*ISD Estimate	Gross Value of All Trades	Total Trading Costs	Net Value of All Trades	No. of Trades Made	Ave. Duration of a position	Net Profit or Loss per trade
EMA5	72737	38239	34498	21	3.6	1643
MAV5	61394	38681	33714	15	2.5	1514
ARMA	70845	40005	30840	21	2.3	1469
LISD	67850	36488	31362	18	2.8	1742
RGN	65175	30047	35128	18	2.5	1952
MEAN	37043	20675	16368	11	9.6	1488

b) 1984

*ISD Estimate	Gross Value of All Trades	Total Trading Costs	Net Value of All Trades	No. of Trades Made	Ave. Duration of a position	Net Profit or Loss per trade
EMA5	4854	(3341)	751	4	9.0	188
MAV5	4079	(1979)	2100	2	5.5	1050
ARMA	6692	(3336)	3356	4	5.0	839
LISD	1380	(1450)	-70	2	7.0	-35
RGN	8615	(3793)	4822	4	5.7	1206
MEAN	1140	(1758)	-607	2	7.0	-304

*Abbreviations are as follows:

EMA5 = 5-day exponentially-weighted moving average of the ISD's,
MAV5 = 5-day equally-weighted moving average of the ISD's,
ARMA = autoregressive-moving 1 day ahead forecasts of the ISD,
LISD = 1-day lag of ISD's (yesterday's value),
RGN = cross-sectional-time-series regression model 1 day ahead forecasts (using an optimal time series predictor of the ISD's from the ARMA modeling),
MEAN = a constant value of the ISD over the entire period equal to an approximation of the mean ISD value for the near-term, at the money option in the preceding period.

Table 8

Average-Absolute and Relative Differences Between Model and Market Prices - By ISD Estimate and Exercise Price

1983

ISD Estimate		June 150	June 165	Sept 145	Sept 150	Sept 160	Sept 165	Sept 165
EMA5	*(a)	.0280	.1008	.2194	.3418	.2484	.2195	.9351
	*(r)	-.0069	.0340	-.0454	-.1048	-.0032	-.1065	-.2839
MAV5	(a)	.0309	.1080	.2287	.3148	.2804	.3099	.9754
	(r)	-.0029	.0558	-.0617	-.1059	-.0091	-.1768	-.5563
ARMA	(a)	.0253	.1242	.2525	.3679	.2850	.1057	.9925
	(r)	-.0119	.0813	-.1141	.1727	.0127	-.0515	-.0783
LISD	(a)	.0259	.1089	.2521	.4599	.2449	.1409	1.1601
	(r)	-.0073	.0155	-.0232	-.0558	-.0011	-.0529	-.1064
RGN	(a)	.0297	.1176	.2214	.3454	.2490	.1960	.8146
	(r)	-.0157	.0740	-.0977	.1614	.1106	.1819	-.0490
MEAN	(a)	.0662	.1152	.5412	.3998	.5783	.6123	-.6328
	(r)	-.0662	.0959	-.5412	-.3909	-.5783	-.6123	.8589

* a = average absolute difference between the Black OPM (BP) and actual market prices (MP), calculated as follows:

$$\sum_{i=1}^{N} |BP_i - MP_P| / N$$

r = average relative difference between the Black OPM and actual market prices, calculated as follows:

$$\sum_{i=1}^{N} (BP_i - MP_i) / N$$

Table 9

Average Absolute and Relative Differences Between Model and Market Prices - by ISD Estimate and Exercise Price

	1984						
ISD Estimate	June 150	June 155	June 160	June 165	Sept 160	Sept 165	Sept 170
EMA5 (a)	.0821	.0851	.0189	.0028	.1727	.0898	.0407
(r)	.0431	.0851	.0189	.0028	.0334	.0131	.0024
MAV5 (a)	.1043	.1203	.0272	.0047	.1469	.0775	.0329
(r)	.0669	.1203	.0262	.0047	.0269	.0099	-.0031
ARMA (a)	.0853	.1318	.0257	.0029	.1761	.0962	.0448
(r)	.0715	.1318	.0193	.0029	.1379	.0543	.0181
LISD (a)	.0555	.0658	.0219	.0023	.0803	.0490	.0319
(r)	.0204	.0556	.0192	.0023	.0058	-.0019	-.0026
RGN (a)	.1249	.1670	.1677	.0041	.1417	.0656	.0403
(r)	.1248	.1670	.0041	.1279	.0260	-.0038	
MEAN (a)	.1410	.4130	.0579	.0037	.1024	.0604	.0635
(r)	.1410	.4130	.0579	.0037	-.0107	-.0429	-.0635